MISEN: A Mobile Indoor White Space Exploration Method

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Abstract—With the exponential growth of the demand for wireless communications, people are faced with the shortage of spectrum resources. To tackle this problem, researchers consider about the further usage of TV channels with low occupancy, referred as white spaces. In this paper, we propose a mobile platform based indoor white space exploration method, namely MISEN, to profile the white space distribution inside a building. In this method, we design a tensor completion based algorithm based on the framework of Alternating Direction Method of Multipliers (ADMM) to recover a complete spectrum map, where we consider the linear dependency of white space information. Moreover, we build a prototype of MISEN and evaluate the performance in real scene. It is shown that MISEN senses on average 18.7% more white spaces with 20.0% less false alarm rate than the state of art methods.

Index Terms—indoor white space, tensor completion, mobile sensing

I. INTRODUCTION

With the rapid growth of wireless communication techniques, the problem of spectrum shortage has become increasingly critical. In 2008, the Federal Communications Commission (FCC) issued a rule, granting unlicensed devices with the access to vacant TV spectrum. Those spectrum is also referred as TV white spaces or simply white spaces. Since then, people have paid more and more attention on the exploration and utilization of TV white spaces.

Many of the existing works [1]–[3] focus on outdoor white space exploration. However, nearly 70% of wireless spectrum usage takes place indoor [4], which makes indoor cases much more important. In this paper, we propose a method to profile the white space distribution inside a building. There exist some efficient methods, such as WISER [5] and FIWEX [6]. Firstly, in those methods, the linear dependency among the temporal dimension is not fully utilized. Secondly, those previous methods require much training for an accuracy sensing. Thirdly, we can further reduce the cost of white space exploration with mobile sensing.

Considering those limitations above, we propose a mobile sensing based indoor white space real-time exploration method, namely MISEN (Mobile Indoor Spectrum ExploratioN). Mobile sensors are used to explore white spaces

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Fig. 1: System architecture of MISEN

inside a building. As a consequence, the number of required sensors will be greatly reduced.

In previous works, it is shown that spectrum information is stable in a short period of time [6]. Motivated by this premise, through combining the latest time slice with historical ones, the dependency among temporal dimension will be considered. In MISEN, under-sampled spectrum data is organized as tensors with time-space-frequency dimensions. An Alternating Direction Method of Multipliers (ADMM) based tensor completion algorithm [7] is designed to recover a complete white space distribution map.

The contributions are summarized below:

- We propose MISEN, a Mobile Indoor Spectrum ExploratioN method. This method achieves high efficiency with extremely low cost. We design a prototype of this mechanism and conduct experiments in real scene. With limited human supervision, the mobile sensors are able to function automatically.
- We design an ADMM based tensor completion algorithm to reconstruct a white space distribution map. The algorithm exploits the historical information and considers the influence of linear dependency among all three dimensions. Our method is the first mechanism to apply ADMM framework in white space exploration.
- We perform a proof-of-concept experiment to evaluate the system performance. Tested within the same data set, MISEN is able to identify more white spaces with less error rate than static sensing methods.

II. OVERVIEW OF MISEN

In this section, we will introduce an overview of MISEN. As shown in Fig.1, MISEN consists of three parts: central host, indoor localization module, and mobile spectrum sensing module. For the first one, while traversing around indoor environments, mobile sensors collect white space information

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which will be transmitted to the central host at regular intervals. At the same time, indoor localization module tracks down the route of mobile sensors and sends them to the host. By combining under-sampled spectrum data with corresponding locations, the central host will reconstruct a white space distribution map. Clients are allowed to acquire white spaces according to their real-time positions.

A. Indoor Localization Module

In MISEN, the indoor localization module is used to obtain real-time positions of clients and record the tracks of mobile sensors, which are sent to the central host in chronological order. In MISEN, we adopt a commercial indoor localization product, which utilizes ultra-wide-band (UWB) signals. Note that the working frequency of this module is 3.8GHz - 5.8GHz, which will not interfere with the wireless signals in TV frequency bands.

B. Mobile Spectrum Sensing Module

For indoor white space exploration, nearly all existing systems utilize static sensing, which is somewhat inefficient due to the failure in exploiting the redundancies in time dimension [6]. Hence, it is reasonable to leverage mobile sensing on the ground to introduce variety in time dimension to spectrum sensing information. Moreover, with mobile sensing, the amount of required sensors will decrease a lot, since a few ones will traverse the whole indoor environment. However, with the reduction of sampling rate, it is even more challenging to reconstruct complete spectrum information. We illustrate the reconstruction algorithm in Section III. Plus, we build a prototype of the mobile spectrum sensing module, by mounting a spectrum sensor and a localization tag on a sweeping robot, as shown in Fig 2. Details of the prototype is presented in Section IV.

Fig. 2: Mobile sensing module

Another challenge is that the collected spectrum information at different channels and positions is asynchronous, due to the mobility of sensors. According to previous work [5], indoor white spaces maintain stable during a short period of time, which stands for the validity of mobile sensing. Hence, MISEN utilizes both real-time data and historical data to solve the data reconstruction problem.

Fig. 3: Incomplete tensor

C. Central Host

The central host is responsible for two tasks: data parsing and spectrum map reconstruction. For the first task, recorded tracks and sampled spectrum information are transmitted to the central host periodically and parsed into an incomplete 3D tensor. For the second task, the host takes the incomplete 3D tensor as input, and then perform a data reconstruction algorithm. The form of input is shown in Fig.3, where the orange blocks represent sampled entries, whereas the grey ones are missing entries. After reconstruction, the latest time slice will be treated as the real-time white space distribution map. Consequently, the central host returns a list of white spaces when receiving requests for white spaces.

In the experiment, the frequency band of concern is 470MHz-806MHz, which is the Chinese digital television band, and the bandwidth of a TV channel is 8MHz. For ease of expression, we use I_1 , I_2 and I_3 to represent the number of channels, time slices and profiled locations respectively. In addition, some important definitions are introduced below.

Ground truth tensor (GT) is a $I_1 \times I_2 \times I_3$ tensor, which contains the ground truth information of signal strengths, denoted by \mathscr{X} .

Loss pattern (LP) is a $I_1 \times I_2 \times I_3$ tensor, which indicates sampled entries, denoted as Ω. For channel *j*, location *i* and time slice *k*,

$$
\Omega(i, j, k) = \begin{cases} 1 & \text{If this entry is sampled,} \\ 0 & \text{Otherwise.} \end{cases}
$$
 (1)

Measurement tensor (MT) is a $I_1 \times I_2 \times I_3$ tensor, which indicates the signal strengths at those sampled entries and 0 at the others, denoted as \mathcal{M} . For channel *j*, location *i* and time slice *k*,

$$
\mathcal{M}(i,j,k) = \begin{cases} \mathcal{X}(i,j,k) & \text{If this entry is observed,} \\ 0 & \text{Otherwise.} \end{cases}
$$
 (2)

Plus, $\mathcal{M} = \Omega \circ \mathcal{X}$, where \circ refers to Hadamard product. **Reconstructed tensor** (RT) is $aI_1 \times I_2 \times I_3$ tensor, which is generated by applying tensor completion algorithm on measured tensor \mathcal{M} , denoted as $\mathcal{\hat{X}}$.

The latest white space distribution is contained in $\mathscr{L}(\cdot, \cdot)$,1) (MATLAB expression). A channel is white space if its signal strength is larger than a *threshold*, and vice versa. In order to protect the licensed user from being interfered, as well as improve the efficiency of data reconstruction, a protection range *PR* is introduced here. Thus, the white space distribution map is defined as following.

$$
MAP(i, j) = \begin{cases} 1 & \text{if } \hat{\mathcal{X}}(i, j, 1) \geq threshold - PR. \\ 0 & \text{if } \hat{\mathcal{X}}(i, j, 1) < threshold - PR. \end{cases}
$$
 (3)

 $MAP(i, j) = 1$ means channel *j* at location *i* is occupied, otherwise vacant.

III. DATA RECONSTRUCTION ALGORITHM

In this section, we will present the tensor reconstruction algorithm. Tensor completion from incomplete measurements has been widely studied [8], [9], and applied to related domains, such as computer vision [7], [10].

Researchers [11] proposed a low rank tensor completion algorithm LRTC and several variations to recover images with random masks. However, we cannot use LRTC in MISEN directly, otherwise the temporal dependency will be neglected. Considering spatial dependence and channel dependence [6], we design a tensor completion algorithm, which is proved to be accurate on exploring indoor white spaces.

A. Notations and Definitions

We use upper case flourish letters for tensors, e.g. \mathscr{X} , upper case letters for matrices, e.g. X and lower case letters for entries, e.g. *x*. An *n*-mode tensor is defined as $\mathscr{X} \in R^{I_1 \times I_2 \times \cdots \times I_n}$, whose entries are expressed as $\mathscr{X}_{i_1,\dots,i_n}$, $1 \leq i_k \leq I_k$, $1 \leq k \leq n$. There are two general operations on tensors: *unfold* and *fold*. The first one converts a tensor into a matrix, $\text{unfold}_k(\mathscr{X}) = X_{(k)} \in R^{I_k \times (I_1 \cdots I_{(k-1)} I_{(k+1)} \cdots I_n)}$. Taking a 3D tensor $\mathscr X$ as an example,

$$
\mathcal{X}_{(1)} = [\mathcal{X}(:,:,1), \mathcal{X}(:,:,2), ..., \mathcal{X}(:,:,I_{3})] \in R^{I_{1} \times (I_{2}I_{3})}
$$

$$
\mathcal{X}_{(2)} = [\mathcal{X}(:,:,1)^{T}, \mathcal{X}(:,:,2)^{T}, ..., \mathcal{X}(:,:,I_{3})^{T}] \in R^{I_{2} \times (I_{1}I_{3})}
$$

$$
\mathcal{X}_{(3)} = [\mathcal{X}(:,:,1)^{T}, \mathcal{X}(:,:,2,:)^{T}, ..., \mathcal{X}(:,:,I_{2},:)^{T} \in R^{I_{3} \times (I_{1}I_{2})}
$$

The opposite operation *fold* converts a tensor into a matrix, $fold_k(X_{(k)}) = \mathcal{X}$, which is the inverse operation of *unfold*. Suppose $X = U\Sigma V^T$, we define a deflation function on *X* as,

$$
\mathbf{F}_{\lambda}(X) = U \Sigma_{\lambda} V^T \tag{4}
$$

where $\Sigma_{\lambda} = \text{diag}(max(\sigma_i - \lambda, 0)), \sigma_i$ is the *i*-th largest singular value of matrix *X*. The trace norm of a matrix *X* is denoted as $||X||_* = \sum_i \sigma_i(X)$. The inner product of matrices is defined as $\langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij}$. The Frobenius norm of a tensor is denoted as $\|\mathscr{X}\|_F = \sqrt{\sum_{ijk} |x_{ijk}^2|}.$

B. Tensor Completion Details

Given Ω and \mathcal{M} , the reconstruction algorithm will return a approximate $\mathscr X$. The principle of this algorithm is to minimize the rank of $\hat{\mathcal{X}}$ while maintaining the known elements. Since the function to solve the rank is non-convex, we use the trace norm [12], which is the closet approximation to the rank.

$$
\hat{\mathcal{X}} = \min_{\mathcal{X}} : ||\mathcal{X}||_*\text{s.t.} : \mathcal{X}_{\Omega} = \mathcal{M}_{\Omega}
$$
\n(5)

However, computing the rank for a tensor with a mode larger than 2 is an NP-hard problem, there is no expression for the convex envelop of the tensor rank. Thus, the definition of tensor trace norm is proposed based on matrix trace norm.

$$
\|\mathcal{X}\|_{*} = \sum_{i=1}^{N} \alpha_{i} \left\|\mathcal{X}_{(i)}\right\|_{*}, \alpha_{i} \geq 0 \tag{6}
$$

where parameters α_i adjusts the weights of each unfolded matrix. With this definition, the problem in (5) will be rewritten,

$$
\min_{\mathcal{X}} : \sum_{i=1}^{N} \alpha_i \| \mathcal{X}_{(i)} \|_{*}
$$
\n
$$
\text{s.t.} : \mathcal{X}_{\Omega} = \mathcal{M}_{\Omega}
$$
\n(7)

To break the dependence among each unfolded matrices, extra matrices M_1, M_2, \ldots, M_n are introduced, where $\mathscr{X}_{(i)} = M_i, 1 \leq i \leq n$. Constraints are further relaxed as $\left\Vert \mathscr{X}_{\left(i\right)}-\mathsf{M}_{i}\right\Vert$ 2 $\frac{f}{f} \leq t_i$. An equivalent formulation with positive parameters β_i is converted from (7).

$$
\mathcal{\hat{L}} = \min_{\mathcal{X}, M_i} : \sum_{i=1}^{n} \alpha_i ||M_i||_* + \frac{\beta_i}{2} ||\mathcal{X}_{(i)} - M_i||_F^2
$$
\n
$$
\text{s.t.} : \mathcal{X}_{\Omega} = \mathcal{M}_{\Omega}
$$
\n(8)

Moreover, considering the linear dependence among three dimensions, the problem is expanded as following,

$$
\mathscr{X} = \min_{\mathscr{X}} : \sum_{i=1}^{n} \alpha_i ||M_i||_* + \frac{\beta_i}{2} ||\mathscr{X}_{(i)} - M_i||_F^2 + ||C_i M_i - C_{i0}||_F^2
$$

s.t. : $\mathscr{X}_{\Omega} = \mathscr{M}_{\Omega}$ (9)

where C_i and C_{i0} represent the constraints for three modes. According to the linear dependence $M_i \approx w_{i_0} + \sum_{k=1}^{K} w_{i_k} M_{i_k}$, $||CM_i - C_0||_F^2$ should be small. The choice of C_i and C_{i0} will be determined on the same way as [6]. An augmented Lagrangian function is defined,

$$
L(\mathscr{X}, M_1, \cdots, M_n, \gamma_1, \cdots, \gamma_n)
$$

=
$$
\sum_{i=1}^n \alpha_i ||M_i||_* + \langle \mathscr{X}_{(i)} - M_i, \gamma_i \rangle + \frac{\beta_i}{2} ||\mathscr{X}_{(i)} - M_i||_F^2
$$
 (10)
+
$$
||C_iM_i - C_{i0}||_F^2
$$

Solving \mathscr{X} . When all other variables are fixed, the optimal $\mathscr X$ is obtained by solving the sub-problem. According to the solution proposed in [13],

$$
\mathscr{X}_{i_1,\dots,i_n} = \begin{cases} \left(\frac{\sum_{i=1}^n \beta_i \text{fold}_i(M_i)}{\sum_{i=1}^n \beta_i}\right)_{i_1,\dots,i_n}, & \Omega_{i_1,\dots,i_n} = 0\\ \mathscr{M}_{i_1,\dots,i_n}, & \Omega_{i_1,\dots,i_n} = 1 \end{cases} \tag{11}
$$

Solving M_i . After obtaining the optimal \mathscr{X} , we can further derive the optimal *Mⁱ* by solving another sub-problem.

$$
\min_{M_i} : \alpha \|M_i\|_{*} + \frac{\beta_i}{2} \|M_i - \mathcal{X}_{(i)}\|_{F}^{2} + \|C_i M_i - C_{i0}\|_{F}^{2} \qquad (12)
$$

The solution to this problem is,

$$
M_i = \mathbf{F}_{\lambda}(\mathscr{X}_{(i)}), \lambda = \frac{\alpha_i}{\beta_i}
$$
 (13)

We apply the framework of Alternating Direction Method of Multipliers (ADMM), which is efficient in solving optimization with multiple non-smooth terms [14]. The parameters will be updated iteratively. A non-negative superscript number with parentheses is used to denote the amount of iterations. For example, $\mathscr{X}^{(k)}$ means \mathscr{X} at iteration *k*.

$$
M_i^{(k+1)} = \arg\min_{M_i} L(\mathcal{X}, M_1^{(k)}, \cdots, M_n^{(k)}, \gamma_1^{(k)}, \cdots, \gamma_n^{(k)})
$$

$$
\mathcal{X}^{(k+1)} = \arg\min_{\mathcal{X} \in P} L(\mathcal{X}, M_1^{(k+1)}, \cdots, M_n^{(k+1)}, \gamma_1^{(k)}, \cdots, \gamma_n^{(k)})
$$

$$
\gamma_i^{(k+1)} = \gamma_i^{(k)} - (M_i^{(k+1)} - \mathcal{X}_{(i)}^{(k)})
$$

where *P* is a convex set, $P = \{ \mathcal{X} \in R^{I_1 \times \cdots I_n} | \mathcal{X}_{\Omega} = \mathcal{M}_{\Omega} \}$ maintaining known elements. The first two steps will be computed according to (13) and (11). The pseudo code of the complete algorithm is shown below:

Algorithm 1 ADMM Based Tensor Completion Algorithm

Input: *M* with $M_{\Omega} = \mathcal{X}_{\Omega}$ and *K* Output: $\mathscr X$ Set $\mathscr{X}_{\Omega} = \mathscr{M}_{\Omega}$ and $\mathscr{X}_{\overline{\Omega}} = 0$ for $k=0$ to K do for $i=0$ to n do $\lambda_i = \frac{\alpha_i}{\beta_i}$ $M_{i} = \mathbf{F}_{\lambda_{i}}(\mathscr{X}_{(i)} + \frac{1}{\beta_{i}})$ β*i* γ*i*) end for $\mathscr{X}_{\overline{\Omega}} = \frac{1}{n} \left(\sum_{i=1}^n \text{fold}_i (M_i - \frac{1}{\beta_i}) \right)$ $\frac{1}{\beta_i} \gamma_i))_{\overline{\Omega}}$ $\gamma_i = \gamma_i - (M_i - \mathcal{X}_{(i)})$ end for return $\mathscr X$

IV. EXPERIMENT SETUP

We build a proof-of-concept prototype of MISEN and conduct experiments in real scene to evaluate the mechanism. In this section, we will introduce the settings of our experiments.

The equipment is grouped into three parts: indoor localization module, mobile sensing module and static sensing module. The localization module is shown in Fig.4. Received positions are transmitted to the computer consistently.

Fig.2 shows the mobile sensing module from different views. From the bottom up, the components are an iRobot Create 2, a USRP N210 [15], a localization tag, a portable battery bank and a log periodic PCB antenna (400-1000 MHz). In the experiment, two mobile sensors are used in total, which helps collect spectrum information from different positions synchronously.

Fig. 4: Indoor localization module

As for the static sensing module, ten sets of hosts and spectrum sensors are deployed to acquire the ground truth information. The timing of start-up across different hosts is synchronized by using the "crontab" command in Ubuntu OS.

We conduct white space measurements on the 3rd floor of our lab building for several weeks. The layout plan of the experiment site is shown in Fig.5, where red, blue and green marks indicate profiling points, static sensing points and localization base points respectively.

Fig. 5: Experiment site

In this experiment, we set the threshold as -83.5 dBm/8 MHz. Furthermore, we set up the gain of the antenna as 0 dBi.

V. PERFORMANCE EVALUATION

We will discuss the performance evaluation of MISEN in this section. We focus on the time span of a tensor, which includes both choices of time slice length and the amount of time slices. Plus, We compare the performance of MISEN with existing systems, including WISER [5] and FIWEX [6]. The experiment settings for those two mechanisms are similar to ours. Please refer to those papers for more information.

A. Experiment Settings

We choose 40 profiled points in the experiment. The white space threshold of signal strength is set up as -83.5dBm/8MHz, which is higher than that in WISER [5]. Although the risk of sensing false white spaces increases, MISEN still achieves a higher accuracy.

For the comparison with existing methods, FA Rate and WS Loss rate are utilized to determine the accuracy and efficiency respectively. These two metrics are defined in [5]:

- False Alarm Rate (FA Rate): the radio between the false vacant channels an all identified vacant channels, including false vacant channels and true vacant channels.
- White Space Loss Rate (WS Loss Rate): the ratio between the false occupied channels and all actually vacant channels, including true vacant channels and false occupied channels.

B. Influence of Tensors' Time Span

In this method, a major factor that influences the performance of MISEN is the total time span *T* of a tensor. With a longer time span *T*, more spectrum information is contained in a tensor. However, we cannot increase the time span of tensor without bound, since larger time span means more data processing time. MISEN would fail to output a white space distribution map in real-time with a tremendous amount of data to process. Thus, in this subsection, we will discuss the influence of the total time span *T* on the performance of MISEN.

Fig. 6: Performance of MISEN with different amount of time slices

The total time span *T* depends on two factors: the amount of time slices, and time slice length. Firstly, we fix the amount of time slices and then change the time slice length. Fig.6 shows the FA Rate and WS Loss Rate respectively for a fixed time slice length. The FA Rate curve goes down rapidly with a small amount of time slices. Then the pace of decline gradually slows down. While the WS Loss Rate curve presents a slower decline.

Secondly, we fix the amount of time slices and then alter the time slice length. Fig.7 illustrates the system performance with different time slice length, where the amount of time slices is set as 4. Both FA Rate and WS Loss Rate go down as the increasing of time slice length. Combining these two factors, larger time spans with richer white space features will leads to a better reconstruction accuracy.

Fig. 7: Performance of MISEN with different time slice lengths

C. Comparison with FIWEX and WISER

Fig. 8: Comparison with WISER and FIWEX

To our best knowledge, WISER and FIWEX are the most lately indoor static sensing methods. For those methods, the system performance is mainly determined by the amount of sensors. For MISEN, the mobile sensing method, the performance lays on the total time span. To compare them fairly, we introduce a concept of data completeness rate, defined by the portion of sampled entries.

The comparison results are illustrated in Fig.8. MISEN outperforms both WISER and FIWEX, which on average detects 18.7% more indoor white spaces with 20.0% less false alarm rate than FIWEX, and 46.1% more white spaces with 64.9% less false alarm rate than WISER. We also conduct data reconstruction on the whole dataset, which lasts for more than 20000 minutes. The time varying performance of these three methods is shown in Fig.9. We can see that False Alarm Rate and White Space Loss Rate of MISEN are lower than static sensing methods steadily.

VI. RELATED WORK

Since FCC granted unlicensed devices with the access to vacant TV channels, people have paid more and more attention to TV white spaces. Researchers studied the white space characteristics on spatial variation, spectrum fragmentation and temporal variation [16]. A lot of outdoor white space measurements were performed in metropolitan cities, such as Chicago [1], Guangzhou [2], Singapore [3].

There are also plenty of works focusing on the applications of TV white spaces. Researchers [17] proposed a protocol for best channel selection from the geo-location database. Based on white space utilization, an enhanced cognitive radio network (E-CRN) [18] was proposed for 5G wireless networks. A scalable sensor network architecture: Sensor Network Over

Fig. 9: Time varying performance of MISEN, FIWEX and WISER

White Spaces (SNOW) was proposed in [19]. Researchers also make efforts to utilize active TV channels, such as WATCH [20], a system to enable WiFi transmission in active TV channels.

For indoor cases, Ying et al. [5] proposed the first indoor white space identification system WISER, applying a channellocation clustering based algorithm. Considering the dependence on both space and frequency, Liu et al. [6] presented FIWEX, a cost-efficient indoor white space exploration mechanism based on compressed sensing algorithm. However, both of them do not consider the temporal dependence. Motivated by this observation, we propose a mobile indoor spectrum exploration mechanism, MISEN.

We use tensor completion method to recover unsampled spectrum data. Tensor completion has been widely studied in the past few years [8], [9], [21], and was applied to lots of fields, including computer vision [7], [10]. Specially, Zhang et al. [7] put forward novel methods for video reconstruction based on tensor-Singular Value Decomposition and tensor nuclear norm.

VII. CONCLUSION

In this paper, we creatively utilize mobile sensing for indoor white space exploration. Furthermore, we propose a mobile indoor white space exploration method, namely MISEN.A proof-of-concept prototype is evaluated in real scene. The results illustrate that MISEN yields a superior performance over existing methods, detecting 18.7% more white spaces with 20.0% less false alarm rate than the state of art solution.

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